CERTIFIED PUBLIC ACCOUNTANT

FOUNDATION LEVEL 1 EXAMINATION

F1.1: BUSINESS MATHEMATICS AND QUANTITATIVE METHODS

DATE: NOVEMBER 2025

MARKING GUIDE AND MODEL ANSWERS

Marking guide

QUESTION ONE *Marks*

a)	Computation with Hurwicz criterion	
	Stating the formula for computing payoffs	0.5
	Calculation of the payoffs for each alternative	
	(0.5 marks each, max 1.5)	1.5
	Determining the best decision	1.0
	Maximum marks	**3.0**
b)	i) Computation of Expected Monetary Value (EMV)	
	Stating the formula for computing EMV	0.5
	Calculation of the EMV for each alternative	
	(0.5 marks each, max 1.5)	1.5
	Determining the best decision	1.0
	ii) Computation of Expected Opportunity Loss (EOL)	
	Stating the formula for computing EOL	0.5
	Calculation of each opportunity loss in the table	
	(0.5 marks each, max 4.5)	4.5
	Calculation of the EMV for each alternative	
	(0.5 marks each, max 1.5)	1.5
	Determining the best decision	0.5
	iii) Drawing the decision tree	
	Drawing each each event node (0.5 each, max 4.5), each decision node and calculation of EMV (0.5 each, max 1.5)	
	(4.5 + 1.5) 6.0	
	Determining the best decision	1.0
	Maximum marks	17
	Total marks	**20**

Model Answer

a) Hurwicz criterion.

	Payoffs are profits				
	(States of nature (Market)				
Decision Alternatives	**Favorable (FRW "000")**	**Moderate (FRW "000")**	**Unfavorable (FRW "000")**	**Maximum**	**Minimum**
Expand	6,500	3,100	(3,900)	6,500	(3,900)
Acquire	8,000	4,500	(5,500)	8,000	(5,500)
No change	4,000	2,000	1,000	4,000	1,000

Payoff, P = α (maximum) + (1 – α) (minimum)
Alternatives
Expand; Payoff = (0.7*6,500,000) + (1– 0.7)*(-3,900,000)
Payoff = FRW 3,380,000

Acquire; Payoff = (0.7*8,000,000) + (1– 0.7)*(-5,500,000)
Payoff = FRW 3,950,000

Make no change; Payoff = (0.7*4,000,000) + (1– 0.7)*(1,000,000)
Payoff = FRW 3,100,000

The best decision under Hurwicz criterion would be to acquire a new facility since it has the highest weighted payoff of FRW **3,950,000**

b) If the probabilities for a favorable market, a moderate market and an unfavorable market are 0.25, 0.40 and 0.35 respectively, then calculate

i) Expected monetary value (EMV)
Expected Monetary Value (EMV) = summation of all outcomes with the probabilities
Expand; EMV = (6,500,000 * 0.25) + (3,100,000 * 0.40) + (-3,900,000 * 0.35) = FRW 1,500,000

Acquire; EMV = (8,000,000 * 0.25) + (4,500,000 * 0.40) + (-5,500,000 * 0.35) = FRW 1,875,000

Make no change; EMV = (4,000,000 * 0.25) + (2,000,000 * 0.40) + (1,000,000 * 0.35) = FRW 2,150,000.

The advice for company using Expected Monetary Value would be to make no change it has the highest expected monetary value of FRW **2,150,000.**

ii) Expected opportunity loss (EOL)

Expected Opportunity Loss Table

Decision Alternatives	Payoffs are profits (States of nature (Market)) Favorable (FRW "000")	Moderate (FRW "000")	Unfavorable (FRW "000")
Expand	8,000-6,500=1,500	4,500-3,100=1,400	1,000- (3,900)=4,900
Acquire	8,000-8,000=0	4,500-4,500=0	1,000- (5,500) =6,500
No change	8,000-4,000 =4,000	4,500-2,000=2,500	1,000-1,000 =0
Probabilities	0.25	0.40	0.35

Solution

Expected Opportunity Loss (EOL) = summation of all outcomes with the probabilities

Expand; EOL = (1,500,000 * 0.25) + (1,400,000 * 0.40) + (4,900,000 * 0.35) = FRW 2,650,000

Acquire; EOL = (0 * 0.25) + (0 * 0.40) + (6,500,000 * 0.35) = FRW 2,275,000

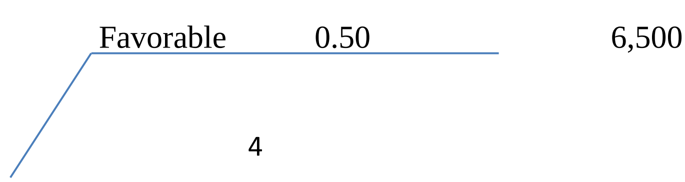
Make no change; EOL = (4,000,000 * 0.25) + (2,500,000 * 0.40) + (0 * 0.35) = FRW 2,000,000

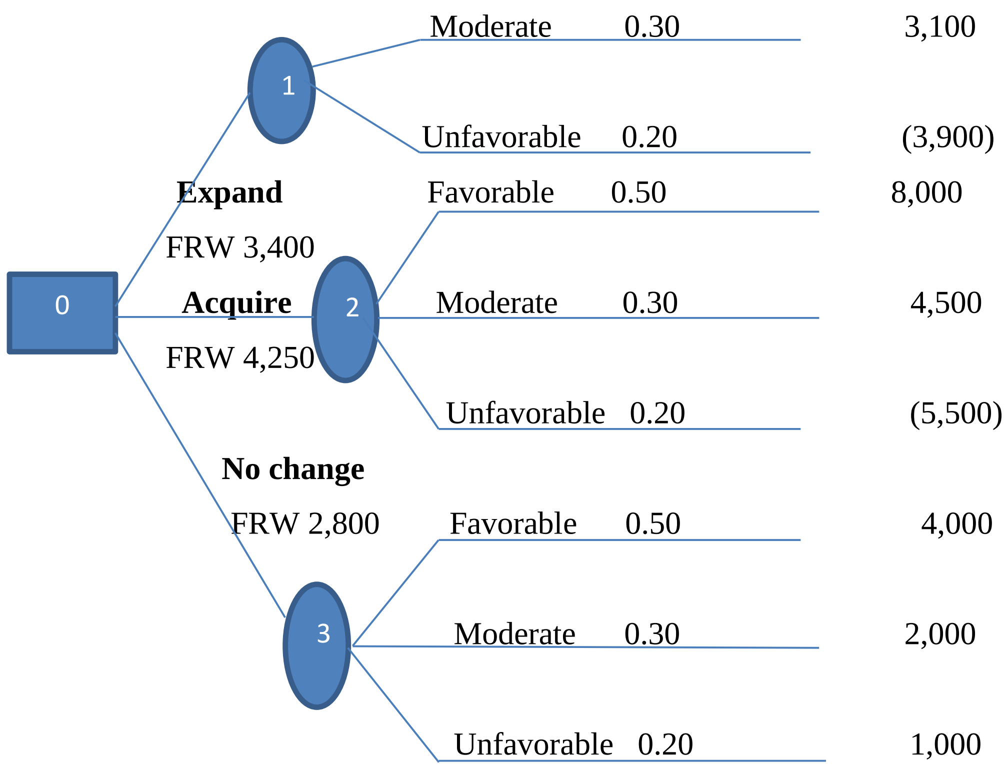
The advice for company using Expected Opportunity Loss would be to make no change because this option has the lowest expected opportunity loss of FRW **2,000,000.**

c) Decision tree

Decision Tree

Probability Payoff (FRW"000")





EMV (FRW "000")

Expand; EMV = (6,500 * 0.50) + (3,100 * 0.30) + (-3,900 * 0.20) = FRW 3,400

Acquire; EMV = (8,000 * 0.50) + (4,500 * 0.30) + (-5,500 * 0.20) = FRW 4,250

Make no change; EMV = (4,000 * 0.50) + (2,000 * 0.30) + (1,000 * 0.20) = FRW 2,800.
Decision: The advice would be to **acquire a new facility as it has highest expected value, FRW 4,500**

QUESTION TWO

Marking guide *Marks*

a)	Finding the derivative	2
	Solving equation	1
	Result	1
	Conclusion	1

b		
	Finding the marginal cost	1
	Finding the marginal revenue	1
	Finding the marginal profit	1
	Finding the value of Finding the marginal cost at x=200	1
	Finding the value of Finding the marginal cost at x=400	1
	Finding the value of Finding the marginal revenue at x=200	1
	Finding the value of Finding the marginal revenue at x=400	1
	Finding the value of Finding the marginal profit at x=200	1
	Finding the value of Finding the marginal profit at x=400	1
c)		
	Finding the breakeven point	2
	Finding the profit	2
	Finding the sales units	2
	Total marks	**20**

Model Answer

a) Given the cost function, $C(x) = 400 - 14x + 0.02x^2$.

The cost is minimized if its first derivative is equal to zero

Derivative of the cost function

$$C'(x) = \frac{d(400-14x+0.02x^2)}{dx} = -14 + 0.04x$$

Formulation of the equation

Marginal cost = 0

$C'(x) = 0 \leftrightarrow -14 + 0.04x = 0$

Solving the equation

It gives $x = \frac{14}{0.04} = 350$

Conclusion

Therefore, to minimize the total cost the complex must have 350 apartments.

b) Given: Total cost $C(x) = 4000 - 32x + 0.08x^2 + 0.00006x^3$
: $Demand\ p(x) = 250 + 0.02x - 0.001x^2$

$$MC = (TC)'$$

$$MR = (TR)'$$

Where $TR = p(x).x$ (demand times output)

$$MP = (TP)'$$

And $TP = TR - TC$

Marginal Cost (MC)

$MC = (4000 - 32x + 0.08x^2 + 0.00006x^3)' = -32 + 0.16x + 0.00018x^2$

For $x = 200$

$MC(200) = -32 + 0.16(200) + 0.00018(200)^2$ = FRW 7,200

For $x = 400 MC(400) = -32 + 0.16(400) + 0.00018(400)^2$ =FRW 60,800 $TR = p(x).x = (250 + 0.02x - 0.001x^2)x = 250x + 0.02x^2 + 0.001x^3$

Marginal Revenue (MR)

$MR = (250x + 0.02x^2 + 0.001x^3)' = 250 + 0.04x - 0.003x^2$

For $x = 200$

MR=250 + 0.04(200) − 0.003(200)^2 = FRW 138,000

For $x = 400$

MR = 250 + 0.04(400) − 0.003(400)^2 = FRW −214,000

Marginal Profit (MP)

$MP = MR - MC$

For $x = 200$

$MP = 138 - 7.2$ =FRW 130,800

For $x = 400$

$MP = -214 - 60.8 = -274.8.$ FRW −274,800

c)

i) $X = \frac{f}{CM} = \frac{f}{P-V} = \frac{800,000}{200-140} = 13,333$ units

Let CM be the contribution margin, $CM = 200 - 140 = 60$

, f fixed cost, therefore, T target profit

ii) $profit = CMx - f = 60(10,000) - 800,000 = FRW - 200,000$

iii) If target profit is FRW 2000000 then the sales $x = \frac{T+f}{CM} = \frac{2,000,000+800,000}{60} = 46,667$units

QUESTION THREE

Marking Guide

	Marks
a) Allocation of 2 marks for each of the four allocation tables and 1 mark for total minimum cost	9
b) Allocate 1 mark for each procedure	5
c)Allocate 2 marks for each method clearly explained	6
Total marks	**20**

Model Answers

a) The following table shows inventory balance at different locations, however, the inventory at Muhanga warehouse need to be calculated using definte integration as follows:

Warehouse	M	K	R
Inventory (Bags)	62	118	120

By using Vogel's Approximation Method, the following procedures are followed:

Step 1: Identify the two lowest costs in each row and column of the given cost matrix and then write the absolute row and column difference. These differences are called penalties.

Step 2: Identify the row or column with the maximum penalty and assign the corresponding cell's min (supply, demand). If two or more columns or rows have the same maximum penalty, then we can choose one among them as per our convenience.

Step 3: If the assignment in the previous satisfies the supply at the origin, delete the corresponding row. If it satisfies the demand at that destination, delete the corresponding column.

Step 4: Stop the procedure if supply at each origin is 0, i.e., every supply is exhausted, and demand at each destination is 0, i.e., every demand is satisfying. If not, repeat the above steps, i.e., from step 1.

Initial Table

Warehouses	**Customers**			
	Southern(S)	Kigali(KI)	Western(W)	**Availability**
Muhanga (M)	50	40	6	62
Kigali (Ki)	70	40	70	118
Rubavu (R)	80	60	70	120
Requirements	140	60	100	300

Table 1 Calculation of penalty and first allocation

Warehouses	**Customers**				**Row penalty**
	Southern(S)	Eastern(E)	Western(W)	**Availability**	
Muhanga (M)	50	40	60	62	**10**
Kigali (K)	70	40(60)	70	118(60)=58	**30**
Rubavu (R)	80	60	70	120	**10**
Requirements	140	60(60)	100	300	
Column Penalty	**20**	**0**	**10**		

The highest penalty is 30 and is found on row number two, the client called Eastern Province (E) will be served by Kigali warehouse (K) a total of 60 bags and only 58 bags remain in Kigali warehouse.

Table 2 Calculation of new penalty and second allocation

Warehouses	**Customers**			**Row penalty**
	Southern (S)	Western(W)	**Availability**	
Muhanga (M)	50(62)	60	62/0	**10**
Kigali (K)	70	70	58	**0**
Rubavu (R)	80	70	120	**10**
Requirements	140/78	100	300	
Column Penalty	**20**	**10**		

Here a customer who has been satisfied is omitted. The highest penalty here is 20 and it is found on the first column. The client Called southern province will be served 62 bags by MUHANGA warehouse and the warehouse will remain empty.

Table 3 Calculation of new penalty and third allocation

	Customers	**Row penalty**

Warehouses	Southern(S)	Western(W)	Availability	
Kigali (K)	70	70	58	**0**
Rubavu (R)	80	70	120	**10**
Requirements	140/78	100	300	
Column Penalty	**10**	**0**		

Here all the penalties are equal, we can select whoever we want to be served first per our convenient. Client southern province (S) will be served remaining 78 bags as follows;58 bags from Kigali wharehouse and 20 bags from Rubavu warehouse while Northern province will be served 100 bags from Rubavu warehouse.

FRW'000'

Total minimum cost= (60*40) + (62*50) + (58*70) + (20*80) + (100*70)

=2400+3100+4060+1600+7000

=18,160 FRW thousands (18,160,000 FRW).

b) **Procedures to be adopted in solving a linear programming problem with graphical method are listed below:**

- Formulation of the appropriate linear programming model
- Graphing the constraints inequalities by treating each inequality as though it was equality and also graph equations by joining two sets of coordinates points.
- Identifying the solution space or the feasible region and shade unwanted space
- Locating the solution points of the feasible region. They occur at the corner points of feasible region.
- Evaluate the objective function at each of the corner points and determine the combination that maximise or minimize the objective function.

c) **North-West Corner Method (NWC)** of solving transportation problems adopt the following procedures:

- Selects the North-west (i.e., upper left) corner cell of the table and allocate the maximum possible units between the supply and demand requirements. During allocation, the transportation cost is completely discarded (not taken into consideration).
- Delete that row or column which has no values (fully exhausted) for supply or demand.

- Now, with the new reduced table, again select the North-west corner cell and allocate the available values.
- Repeat steps (ii) and (iii) until all the supply and demand values are zero.
- Obtain the initial basic feasible solution.

On the other hand, **Vogel's approximation method** (VAM) and **least cost method** adopts these procedures:

Algorithm for Least Cost Method (LCM)

- Select the smallest transportation cost cell available in the entire table and allocate the supply and demand.
- Delete that row/column which has exhausted. The deleted row/column must not be considered for further allocation.
- Again, select the smallest cost cell in the existing table and allocate. (Note: In case, if there are more than one smallest costs, select the cells where maximum allocation can be made)
- Obtain the initial basic feasible solution.

Algorithm for Vogel's Approximation Method (VAM)

- Calculate penalties for each row and column by taking the difference between the smallest cost and next highest cost available in that row/column. If there are two smallest costs, then the penalty is zero.
- Select the row/column, which has the largest penalty and make allocation in the cell having the least cost in the selected row/column. If two or more equal penalties exist, select one where a row/column contains minimum unit cost. If there is again a tie, select one where maximum allocation can be made.
- Delete the row/column, which has satisfied the supply and demand.
- Repeat steps (i) and (ii) until the entire supply and demands are satisfied.
- Obtain the initial basic feasible solution.

QUESTION FOUR

Marking Guide	***Marks***
a) (i) Advantages (1 mark for each, max 3)	3
Disadvantages (1 mark for each, max 2)	2
ii) Limitations for systematic sampling (mark1*2	2
b) i) Formula of median	1
Application of the formula	1
ii) Draw bar chat	
(1 mark for labeling x axis and 1 mark labeling y axis)	2
(0.5 marks for each bar drawn, max 2.5)	2.5
c) Standard deviation	
Calculation of the square of deviations (0.5 marks for each, max 4.5)	4.5
Formula of standard deviation	1.0
Result	1.0
TOTAL	**20**

i) Advantages of stratified sampling techniques

Stratified sampling is a sampling technique commonly used in research and surveys to ensure that the sample accurately represents the population being studied. It involves dividing the population into distinct subgroups or strata based on certain characteristics, and then selecting a proportionate number of participants from each stratum. This method offers several advantages and disadvantages, which are discussed in detail below.

Advantages of stratified sampling techniques:

1. **Increased precision**: One of the main advantages of stratified sampling is that it can lead to increased precision in estimating population parameters. By dividing the population into homogeneous subgroups, the variability within each stratum is reduced. This allows for more accurate estimates of the population characteristics, as the sample is more representative of the entire population.

2. **Improved representativeness**: Stratified sampling ensures that each subgroup or stratum is represented in the sample, which helps to capture the diversity within the population. This is particularly useful when studying populations with distinct characteristics or when there are significant differences between subgroups. By including participants from each stratum, stratified sampling provides a more comprehensive understanding of the population as a whole.

3. **Efficient resource utilization**: Another advantage of stratified sampling is that it allows for efficient use of resources. By focusing on specific subgroups within the population, researchers can allocate their resources effectively and obtain reliable results without having to survey the

entire population. This can save time, money, and effort compared to other sampling techniques.

Disadvantage of stratified sampling techniques:

1. **Complexity in implementation**: Stratified sampling requires careful planning and execution. Researchers need to identify relevant characteristics or variables to define the strata, which can be challenging depending on the nature of the study. Additionally, determining appropriate sample sizes for each stratum requires knowledge about the distribution of these characteristics within the population.
2. **Potential for selection bias:** While stratified sampling aims to improve representativeness, there is still a risk of selection bias if the stratification variables are not chosen correctly or if the sampling process is flawed. If certain subgroups are underrepresented or excluded from the sample, the results may not accurately reflect the population as a whole. It is crucial to ensure that the stratification variables are relevant and that the sampling process is unbiased.

Limitations of systematic sampling

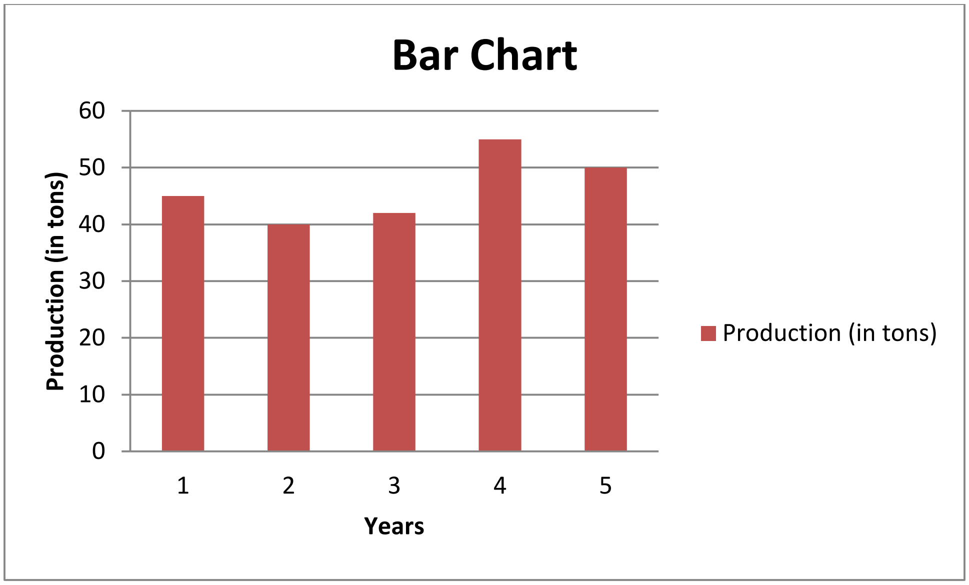
- Risk of bias: While there's a low risk of bias, there's still a risk that has to be managed. ...
- Risk of data manipulation: When you use systematic sampling, you're setting up a system to use. ...
- Requires population size: The other risks can be controlled for, but this disadvantage is an inherent one.

Class	0-20	20-40	40-60	60-80	80-100	100-120	120-140	140-`160	Total
Frequency=f	10	15	15	20	8	8	6	8	90
Cumulative frequency	10	25	40	60	68	76	82	90	

$M_e = L_0 + \frac{h}{f_0}\left(\frac{n}{2} - F\right)$

$L_0 = 60$, $h = 20$, $f_0 = 20$, $\frac{n}{2} = 45$, $F = 60$ therefore, $M_e = 60 + \frac{20}{20}(45 - 60) = 65$

b) The bar chart for the data is as follow



Bar chart for the data in Table2

c) By assumed mean method

questions yet to be completed are: 28, 25, 23, 30, 27, 24, 25, and 23

Assumed mean $A = 25$

Items:Xi	Deviation:di=xi-A	di*di
23	-2	4
23	-2	4
24	-1	1
25	0	0
25	0	0
27	2	4
28	3	9
30	5	25
Sum	5	47

$$sd = \sqrt{\frac{\sum {d_i}^2}{n} - (\frac{\sum d_i}{n})^2} = \sqrt{\frac{47}{8} - \frac{25}{64}} = \sqrt{\frac{351}{64}} = 2.34$$

QUESTION FIVE	
Marking guide	Marks
a) Price indices	

Presentation of data in a table	1
Totals (0.5 marks for each, max 2)	2
Computation of the price indices (1 mark for each, max 3)	3
Interpretation (1 mark for each, max 3)	3
b)i) Allocate 6 marks for a network diagram (0.5 marks for each activity, max 6),0.5 marks for critical path and 0.5 marks for project completion time.	7
b)ii)Allocate 2 marks for a definition and 1mark for each benefit	4
Total marks	**20**

Model Answers

a) i)Laspeyre's price index

Laspeyre's price index $= \frac{\sum P_1Q_0}{\sum P_0Q_0} \times 100 = \frac{80000}{98750} \times 100$= 81.01%

Paasche's price index

Paasche's price index $= \frac{\sum P_1Q_1}{\sum P_0Q_1} \times 100 = \frac{88900}{111450} \times 100$ = 79.76%

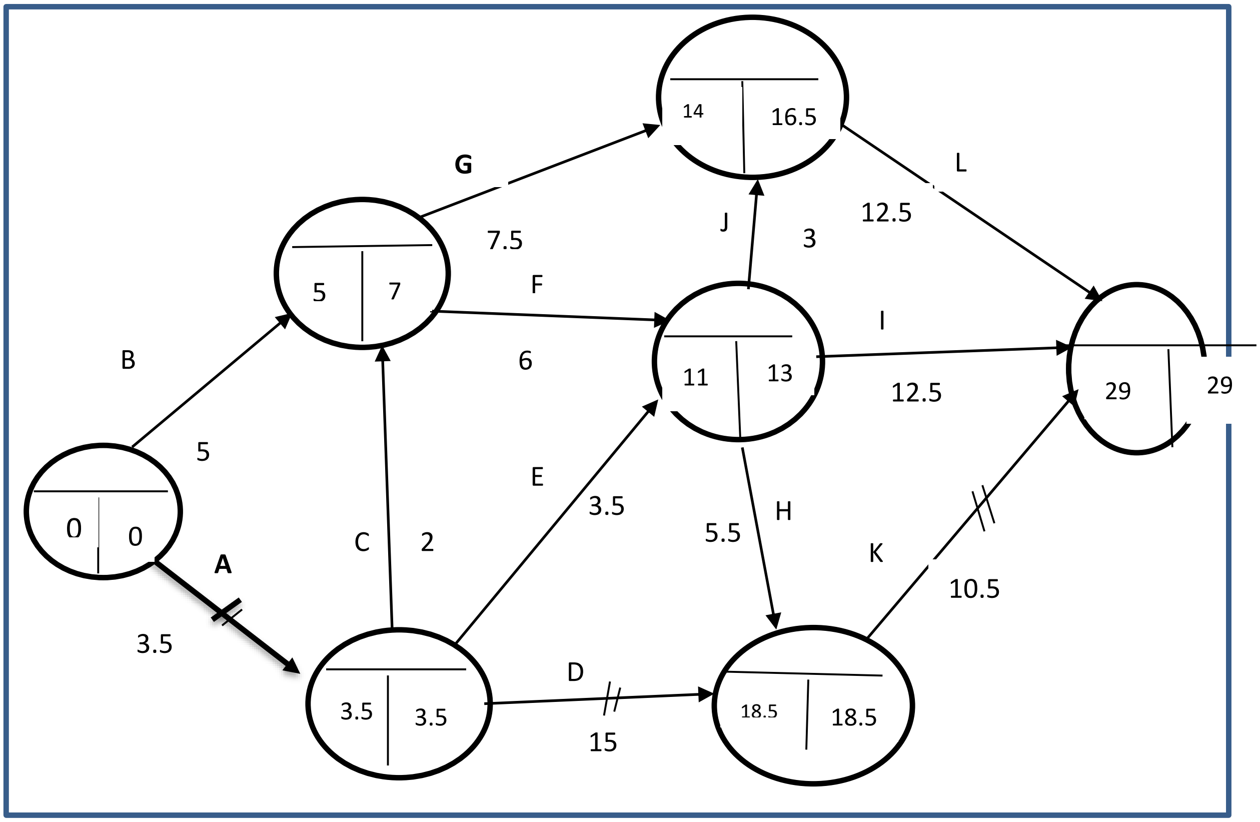
Marshall Edge Worth's price index

Marshall Edge Worth's price index $= \frac{\sum P_1Q_0 + \sum P_1Q_1}{\sum P_0Q_0 + \sum P_0Q_1} \times 100 = \frac{80000 + 88900}{198750 + 111450} \times 100$ = 80.35%

	Base year			Current year				
Commodity Item	**Price (FRW)/Kg**	**Quantity (Kgs)**	**Po*Qo**	**Price (FRW)/Kg**	**Quantity (Kgs)**	**P1*Q1**	**P1*Qo**	**P0*Q1**
Rice	400	30	12000	700	30	21000	21000	12000
Potatoes	175	50	8750	100	54	5400	5000	9450
Flour	1000	24	24000	750	30	22500	18000	30000
Wheat	1500	36	54000	1000	40	40000	36000	60000
Total			**98,750**			**88,900**	**80,000**	**111,450**

a) A network diagram:

Activity	A	B	C	D	E	F	G	H	I	J	K	L
Predecessor(s)			A	A	A	B, C	B, C	E, F	E, F	E, F	D, H	G, J
Completion time(hours)	3.5	5	2	15	3.5	6	7.5	5.5	12.5	3	10.5	12.5



The anticipated project completion time is **29 months** and its critical path is **ADK**

Activity G can be delayed by 4 months while activity L can be delayed by 2.5 months without delaying the total project completion time.

b) **Gantt Chart** is a visual representation of tasks plotted against time. It represents crucial information in a projects such as who is assigned to what and task duration.

Benefits of Gantt chart

1. It allows better tracking of projects. It shows tasks, milestones, potential constraints and overall workflow. This enables project leaders to make appropriate adjustments.
2. A gantt chart boosts productivity through enabling workers to collaborate in order to increase production. Great visibility helps workers to keep focused on the tasks they need to finish.
3. It provides high level overview. A gantt chard provide the project perspective and its timeframe

4. It illustrates overlaps and dependencies between tasks
5. The chart helps to manage complex tasks as it depicts tasks in a simplest manner
6. The chart lists all the tasks to be performed for the completion of a project and this facilitate the team to comply with deadlines
7. It keeps everyone aligned including remote workers since it enhances communication and teamwork

QUESTION SIX

Marking Guide	***Marks***
a) Equation of trend line formula	0.5
Summations of x, y x2 and xy (0.5 marks for each, max 2)	2
Formula and computation of a (0.5 marks for each, max 1)	1
Formula and computation of b (0.5 marks for each, max 1)	1
Equation of trend line	0.5
Trend values (one mark to each value)	5
c) b) i) State Hawkins-Simon condition	0.5
Verification of Hawkins-Simon condition	1
Conclusion	0.5
ii) Formula	**1**
Calculation (application 2 result 1)	3
iii) Formula	1
Calculation (application 2 result 1)	3
TOTAL	**20**

a) The equation of the trend line is $y = a + bx$

$$\sum y = na + b\sum x$$

$$\sum xy = a\sum x + b\sum x^2$$

$$mean = \frac{sumx}{n} = \frac{2000 + 2001 = 2002 + 2003 + 2004}{5} = 2002$$

Year (X)	Sales(Y)	x =X-Mean	$\bar{X}^2$	XY
2000	35	-2	4	-70
2001	56	-1	1	-56
2002	79	0	0	0
2003	80	1	1	80
2004	40	2	4	80
sum	290	0	10	34

Now $290 = 5a + b(0) \leftrightarrow 290 = 5a \leftrightarrow a = \frac{290}{5} = 58$

$$34 = a(0) + b(10) \leftrightarrow 34 = 10b \leftrightarrow b = \frac{34}{10} = 3.4$$

Substituting these values in the equation of trend line which is $y = 58 + 3.4x$ with 2002=0 we have:

Year	x-2002	Trend value
2000	-2	58+3.4(-2) =51.2
2001	-1	58+3.4(-1) =54.6
2002	0	58+3.4(0) =58
2003	1	58+3.4(1) =61.4
2004	2	58+3.4(2) =64.8

b) Here the technology matrix is given as:

	Steel	Coal	Final demand
Steel	0.4	0.1	50
Coal	0.7	0.6	100
Labour	5	2	

$I - A = \begin{pmatrix} 0.6 & -0.1 \\ -0.7 & 0.4 \end{pmatrix}$

i) The system will be viable if Hawkins-Simon condition are satisfied:

- the diagonal element of $(I - A)$ are all positive.

In present problem it is: 0.6 > 0

0.4 > 0

- the determinant should be positive.

In present problem it is: (0.6 * 0.4) – (-0.7 *- 0.1) = 0.24 - 0.07 = 0.17 > 0

Hence, the system is viable.

ii) The required gross output is given by:

$X = (I - A)^{-1}F = 1/0.17 \begin{pmatrix} 0.6 & -0.1 \\ -0.7 & 0.4 \end{pmatrix}^{-1} \begin{pmatrix} 50 \\ 100 \end{pmatrix}$

$= \frac{1}{0.17} \begin{pmatrix} 30 + 10 \\ 35 + 40 \end{pmatrix} = \begin{pmatrix} 235.29 \\ 41.18 \end{pmatrix}$

Total labour day required are 5 * steel output + 2 * coal output

$= 5 * 176.5 + 2 * 558.8 = 882.5 + 1117.6 = 2000\ labour\ day$

iii) From the text in b) $A = \begin{pmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{pmatrix}$

$$I - A = \begin{pmatrix} 0.6 & -0.1 \\ -0.7 & 0.4 \end{pmatrix}$$

$$(I - A)' = \begin{pmatrix} 0.6 & -0.7 \\ -0.1 & 0.4 \end{pmatrix}$$

$$\begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} 0.6 & -0.7 \\ -0.1 & 0.4 \end{pmatrix}^{-1} \begin{pmatrix} 5 * 100{,}000 \\ 2 * 100{,}000 \end{pmatrix} = \frac{1}{0.17} \begin{pmatrix} 0.4 & 0.7 \\ 0.1 & 0.6 \end{pmatrix} \begin{pmatrix} 500{,}000 \\ 200{,}000 \end{pmatrix}$$

$$= \frac{1}{0.17} \begin{pmatrix} 200{,}000 + 140{,}000 \\ 50{,}000 + 120{,}000 \end{pmatrix}$$

$$P_1 = Steel = FRW\ 2{,}000{,}000\ per\ tonne$$
$$P_2 = Coal = FRW\ 1{,}000{,}000\ per\ tonne$$

QUESTION SEVEN

Marking Guide	***Marks***
a) i) Determination of consumption matrix C	2
ii) Stating the Leontief input – output equation	1
iii) Computation of Leontief matrix	1
Computation of the determinant of a matrix	1
Computation of adjoin	1
Computation of production units	3
b)	
i) Definition 1 mark at each	**2**
ii) Formula, computation and result 1 mark each	3
iii) Formula	**2**
Substitution and Calculation	2
Result	2
TOTAL	**20**

Model Answer

a)

i) Consumption matrix C

	To	
From	Farming (Units)	Fishing (Units)
Farming (Units)	0.6	0.4
Fishing (Units)	0.3	0.3

Let x_1 represent farming sector and x_2 represent fishing sector

Consumption matrix, $C = \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.3 \end{bmatrix}$

ii) Leontief input – output equation

X = CX + D

Let x_1 represent farming sector and x_2 represent fishing sector

X represents x_1 and x_2 $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

C represents consumption matrix; $C = \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.3 \end{bmatrix}$

D represents the demand; $D = \begin{bmatrix} 2{,}000 \\ 4{,}000 \end{bmatrix}$

iii) Inverse matrix

First find the inverse matrix from X = CX + D

X – CX = D

X (I – C) = D

$X = (I - C)^{-1}D$

Then solve for the inverse matrix $(I - C)^{-1}$

$(I - C)^{-1} = \frac{(I - C)^T}{\det (I - C)}$

Compute the Leontief matric (I – C)

$C = \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.3 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Therefore $(I - C) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.3 \end{bmatrix} = \begin{bmatrix} 0.4 & -0.4 \\ -0.3 & 0.7 \end{bmatrix}$

Then the determinant of (I – C) is given by: (

(I – C) = $\begin{bmatrix} 0.4 & -0.4 \\ -0.3 & 0.7 \end{bmatrix}$

det (I – C) = (0.4 * 0.7) – (- 0.3 * - 0.4) = 0.16

Determine the adjoint $(I - C)^T$

$(I - C)^T = \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.4 \end{bmatrix}$

Solve for the inverse

$(I - C)^{-1} = \frac{(I - C)^T}{\det (I - C)} = \frac{1}{0.16} \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.4 \end{bmatrix}$

Finally solve for the production units required to satisfy the demand given

$X = (I - C)^{-1}D$

$(I - C)^{-1} = \frac{1}{0.16} \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.4 \end{bmatrix}$ D = $\begin{bmatrix} 2{,}000 \\ 4{,}000 \end{bmatrix}$

$X = \frac{1}{0.16} \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.4 \end{bmatrix} * \begin{bmatrix} 2{,}000 \\ 4{,}000 \end{bmatrix}$

$X = \frac{1}{0.16} \begin{bmatrix} (0.7 * 2{,}000) & (0.4 * 4{,}000) \\ (0.3 * 2{,}000) & (0.4 * 4{,}000) \end{bmatrix} = \frac{1}{0.16} \begin{bmatrix} 1{,}400 + 1{,}600 \\ 600 + 1{,}600 \end{bmatrix}$

$X = \frac{1}{0.16} \begin{bmatrix} 3{,}000 \\ 2{,}200 \end{bmatrix} = \begin{bmatrix} 18{,}750 \\ 13{,}750 \end{bmatrix}$

$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 18{,}750 \\ 13{,}750 \end{bmatrix}$.

The production necessary to satisfy the final demand is 18,750 units from farming sector and 13,750 units from fishing sector.

b)

i) Different between permutation and combination

Permutation: arrangements where order matters.

$${}^{n}P_r = \frac{n!}{(n-r)!}$$

Combination: selections where order does not matter.

$$^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

ii) Number of different ways to arrange 5 students by using the permutation

$$^{5}P_{5} = 5! = 120$$

iii) Determine the total number of distinct ways in which such a sub-committee

The designated person must be the chair and must be included, so choose the remaining 5 members from the other 11:

$$^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$^{11}_{5}C = \frac{11!}{5!(11-5)!} = \frac{11!}{5!6!}$$

$$^{11}_{5}C = \frac{11!}{5!6!} = \frac{11*10*9*8*7*6!}{120*6!} = \frac{55{,}440}{120}$$

$$^{11}_{5}C = 462$$